LUPINE PUBLISHERS

Journal of Mining and Mechanical Engineering

DOI: 10.32474/JOMME.2022.01.000124

ISSN: 2690-5779

Short Communication

Energy Approach to Convergence Acceleration of Step-By-Step Iterative Methods in Finite Element Analysis of Geometrically Nonlinear Structures

Vladimir P Agapov*

Department of Civil Engineering, National Research Moscow State University, Russia

***Corresponding author:** Department of Civil Engineering, National Research Moscow State University, Russia

Received: December 14, 2021 **Published:** January 25, 2022

Short Communication

One of the problems arising in the analysis of nonlinearly deformable structures by

the finite element method is the acceleration of the convergence of step-iterative procedures. In this paper, a method for convergence accelerating is considered, based on the use of both static and energy correction of the solution.

Equilibrium equations of a structure for a load step can be written in the form [1]:

$$\left[K + K_u + K_\sigma + K_{NL} + K_{NL_2}\right]\{\Delta u\} = \{\Delta P\}. \qquad (1)$$

where $[K],\left[K_{NL_1}\right],\left[K_{NL_2}\right]$ - the stiffness matrices of the zero, first and second orders $[K_u],[K_\sigma]$ - the matrices of initial displacements and stresses $\{\Delta u\}$ - the vector of increments of nodal displacements $\{\Delta P\}$ - the vector of increments of nodal forces, respectively.

Matrices for an individual finite element are written as follows [2]:

$$\left[K_{NL_i}\right] = \left[\overset{\circ}{K}_{NL_i}\right] + \left[K^d_{NL_i}\right]$$
$$\left[K^d_{NL_i}\right] = 0.5\left[\{\Delta u\}^T \frac{\partial}{\partial(\Delta u_j)}\left[\overset{\circ}{K}_{NL_i}\right]\right] \quad i = 1,2;\ j = 1,2,\ldots,n \qquad (2)$$

Where $\left\lfloor \overset{\circ}{K}_{NL_i} \right\rfloor$ and $\left\lfloor K^d_{NL_i} \right\rfloor$ are the direct and differential stiffness matrices of the i-th order, n is the number of degrees of freedom of a finite element.

Equation (2) means that the j-th row of the matrix can be found as the product of the transposed vector of nodal displacements by the first derivative of $\left\lfloor \overset{\circ}{K}_{NL_i} \right\rfloor$ with respect to the j-th degree of freedom.

When solving a nonlinear static problem in increments and using modified Lagrangian coordinates, equation (1) is written in the form:

$$\left\lfloor K + K_\sigma + K_{NL_1} + K_{NL_2} \right\rfloor \{\Delta u\} = \{\Delta p\}, \qquad (3)$$

We will solve equation (3) by an iterative method of additional loading, which is equivalent to using the modified Newton-Raphson method:

$$\left\lfloor K + K_\sigma \right\rfloor_j \{\Delta u\}^{(i)}_j = \{\Delta P\}_j - \left[K_{NL_1} + K_{NL_2}\right]^{(i-1)}_j \{\Delta u\}^{i-1}_j, \qquad (4)$$

where j - the number of the loading step, i - the iteration number at this step.

To accelerate the convergence of the iterative process, we use the energy relations. The displacement vector determined from the equilibrium equations (4) must also satisfy the energy conservation law. For each loading step, you can write

$$W_{j-1,J-1} + W_{j-1,j} + W_{J,J} + A_{j-1,j-1} + A_{j-1,j} + A_{j,j} = 0, \qquad (5)$$

where $W_{j-1,j-1}, A_{j-1,j-1}$ - the work of external and internal forces of the initial state on initial displacements, $W_{j-1,j}, A_{j-1,j}$ - the work of external and internal forces of the initial state on additional displacements $\{\Delta u\}_i, W_{j,j}, A_{j,j}$ - the work of additional external and internal forces on additional displacements.

For the initial state, the energy conservation law is observed. Hence

$$W_{j-1,j-1} + A_{j-1,j-1} = 0, \qquad (6)$$

Since the initial state is in equilibrium, then, considering the additional displacements to be sufficiently small, based on the principle of possible displacements, we can write

$$W_{j-1j} + A_{j-1j} = 0 \qquad (7)$$

Considering (5) - (7), we obtain:

$$W_{J,J} + A_{j,j} = 0. \qquad (8)$$

We require that the solution obtained from (4) at the i-th iteration satisfies relation (7), i.e.

$$W_{j,j}^{(i)} + A_{j,j}^{(i)} = 0 \qquad (9)$$

To achieve this, we introduce the correction factor "c" as follows:

$$\left\{\Delta\bar{u}\right\}_j^i = c\left\{\Delta u\right\}_j^i. \qquad (10)$$

Let's calculate the work of internal forces. As shown in [1]:

$$A_{j,j} = A_{J,J}^{(2)} + A_{J,J}^{(3)} + A_{J,J}^{(4)} \qquad (11)$$

Where

$$\left.\begin{aligned} A_{J,J}^2 &= -\frac{1}{2}\left\{\Delta\bar{u}\right\}_j^T [K]\left\{\Delta\bar{u}\right\}_j - \frac{1}{2}\left\{\Delta\bar{u}\right\}_j^T [K_\sigma]\left\{\Delta\bar{u}\right\}_j; \\ A_{J,J}^3 &= -\frac{1}{2}\left\{\Delta\bar{u}\right\}_j^T \left[K_{NL_1}\right]\left\{\Delta\bar{u}\right\}_j; A_{J,J}^4 = -\frac{1}{2}\left\{\Delta\bar{u}\right\}_j^T \left[K_{NL_2}\right]\left\{\Delta\bar{u}\right\}_j. \end{aligned}\right] \qquad (12)$$

The work of external forces can be found like this:

$$W_{j,j} = \frac{1}{2}\left\{\Delta\bar{u}\right\}_j^T \{\Delta P\}_j. \qquad (13)$$

Using relations (10) - (13), we arrive at the algebraic equation:

$$b_0 + b_1 c + b_2 c^2 + b_3 c^3 = 0 \qquad (14)$$

whose coefficients are equal:

$$\left.\begin{aligned} b_0 &= \frac{1}{2}\left\{\Delta\bar{u}_j^{(i)}\right\}^T \{\Delta P\}_j; b_1 = \frac{1}{2}\left\{\Delta\bar{u}_j^{(i)}\right\}^T [K + K_\sigma]\left\{\Delta\bar{u}_j^{(i)}\right\} \\ b_0 &= \frac{1}{2}\left\{\Delta\bar{u}_j^{(i)}\right\}^T \left[K_{NL_1}\right]\left\{\Delta\bar{u}_j^{(i)}\right\}, b_3 = \frac{1}{2}\left\{\Delta\bar{u}_j^{(i)}\right\}^T \left[K_{NL_2}\right]\left\{\Delta\bar{u}_j^{(i)}\right\}. \end{aligned}\right]$$

Equation (14), as a rule, has one positive root. However, if there are several such roots, then you need to choose the closest to one in order to provide the smallest number of equilibrium iterations.

The described above method of convergence acceleration was realized in computer program PRINS [3]. The effectiveness of the method has been proven in practice.

References

1. Agapov VP (1984) Osnovnye sootnosheniya MKE v staticheskih i dinamicheskih raschetah geometricheski nelinejnyh konstrukcij // Stroitel'naya mekhanika i raschet sooruzhenij. - N 5 C pp. 43-47.
2. Agapov VP Uchet geometricheskoj nelinejnosti v staticheskih i dinamicheskih raschetah konstrukcij metodom konechnyh elementov// Uchenye zapiski CAGI (5): 90-98.
3. Agapov VP (2005) Metod konechnyh ehlementov v statike, dinamike i ustojchivosti konstrukcij, Izd-vo ASV, Moscow, Russia.

DOI: 10.32474/JOMME.2022.01.000124