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Review Article

The Scaling Law for Resistance of a Wearable Fabric Sensor: Fractal Analysis and Experimental Verification

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Abstract

The resistance formulation for a metal conductor is well-known to all students. It scales with the ratio of its length to the square of the radius. For non-metal conductors, e.g., a wearable fabric sensor, this formulation becomes totally invalid, and a fractal modification has to be adopted. This paper employs the two-scale fractal theory to establish a scaling law for the resistance of a carbon-based sensor material for smart fabric applications, and an experiment is carried out to verify the new formulation.

Keywords: Graphene-based Composites; Two-scale Fractal Dimension; Scaling Law; Carbon

Introduction

The resistance property of a conductive wearable fabric sensor is the most important in smart fabric applications, especially in continuous health monitoring [1]. The resistance calculation is far different from that for a conductor. A conductor is the cornerstone in the fundamental theory of circuit and its resistance can be calculated by the following formulation

$$R = \frac{k\lambda}{\pi r^2} \quad (1)$$

where R, λ, and r, respectively, the resistance, the length and the radius of the resistor, k is a material constant. Eq. (1) can be written in two scaling laws:

$$R \sim \lambda^1 \quad (2)$$

and

$$R \sim 1/r^2 \quad (3)$$

The scaling law of Eq. (2) can be explained as the straight-line transmission, see Figure 1(B), the exponent of 1 implies the dimension of the straight line. The scaling law of Eq. (3) implies that electrons are occupied on the whole section with the dimension of 2, see Figure 1(A).

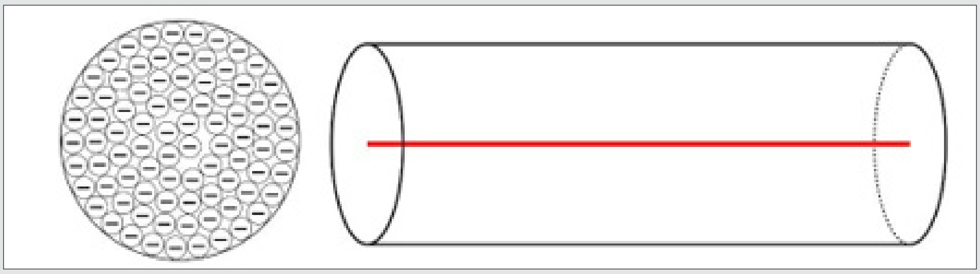

Figure 1: Resistance for a metal conductor.

Scaling Laws For Carbon-Based Materials

Eq. (1) is valid for metal resistors, for non-metal ones, Chinese mathematician, Dr. Ji- Huan He, suggested a scaling law to calculate the resistance [2-4]:

$$R = \frac{k\lambda^{\alpha}}{r^{\beta}} \quad (4)$$

where α is the fractal dimension of the electron trajectory as illustrated in Figure 2, β is the fractal dimension of the section, α=1 for a metal conductor, see Figure 1(B). We assume that there are many electrons occupied in the section, as illustrated in Figure 1(A), the continuum section for a metal conductor leads to β=2. For the surface resistance like that in the electrospinning process, β=1, see Figure 3; Eq. (4) can be written in two scaling laws:

$$R \sim \lambda^{\alpha} \quad (5)$$

and

$$R \sim 1/r^{\beta} \quad (6)$$

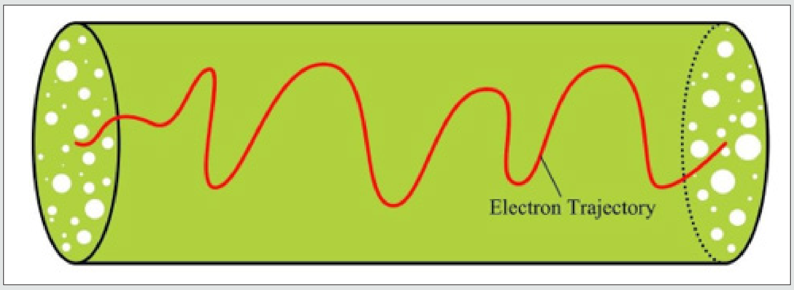

Figure 2: The transmission trajectory of charges.

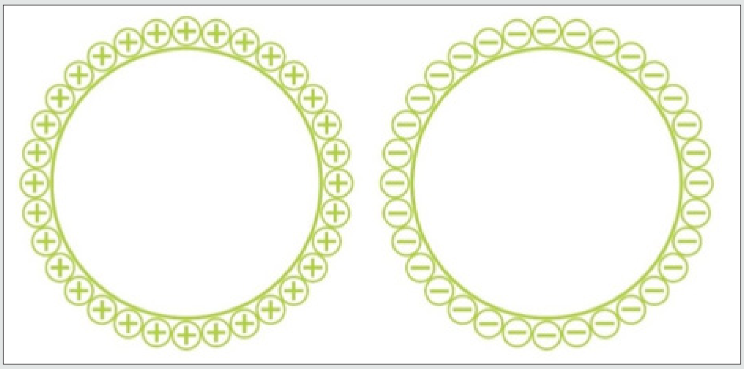

Figure 3: Resistance for surface convection.

The scaling law of Eq. (5) can be explained as the non-straight-line transmission of charges, the exponent of α is the two-scale fractal dimensions of the curve line of the charge transmission, see Figure 2. The scaling law of Eq. (6) implies that electrons are occupied partly on the section with the two fractal dimensions of β. Consider a charged jet in an electrospinning process, see Figure 3, the current due to the surface convection of the charges can be written as [5,6]

$$I = 2\pi r\zeta w \quad (7)$$

where I am the surface current, ζ is the charge density per length, w is the charged jet's velocity. When the surface charge is negative, the current direction follows the charged jet's motion, however, when the surface charge is positive, the current direct is opposite the jet's motion. Eq. (7) implies that

$$R=(U/I) \sim (1/r) \quad (8)$$

where U is the applied voltage on the charged jet. Eq. (8) meets the scaling law of Eq. (6), for the section is two dimensional and its boundary of its perimeter is one dimensional (Figure 4).

Two-Scale Fractal Dimension

The formulation for calculation of the two-scale fractal dimensions is as follows [7-10]

$$\alpha = \alpha_0 \times \frac{M}{N} \quad (9)$$

where M/N is ratio of the measured units under two scales, and α0 is dimensions for large scale. For example, the two-scale fractal dimensions for the Koch curve is 4/3=1.1333 instead of the Hausdorff fractal dimension of ln4/ln3=1.2618. The Hausdorff fractal dimension is for strict fractal patterns, while the two-scale fractal dimension is used to figure out the porosity and unsmooth boundary. The two-scale fractal theory has been widely used as an effective mathematics tool to analysis of various discontinuous problems, for examples, the fractal rheological law [11,12] the fractal oscillator for 3D printing technology [13], the fractal mechanical and electrical properties of graphene/sic composites [14,15], the time-fractional Kundu- Mukherjee-Naskar equation [16], the fractal diffusion law [17], the fractal two-phase flow model18, the fractal convection-diffusion law, and Fangzhu's water collection from air [19-22].

In this paper we will study the scaling law for resistance of a lead of a pencil, which consists mainly of graphene. Consider a unit of the graphene as illustrated in Figure 4(B), we have M/N=5/4 and $a_0 = 2$, so we have

$$\alpha = 2\times\frac{5}{4} = 2.5 \quad (10)$$

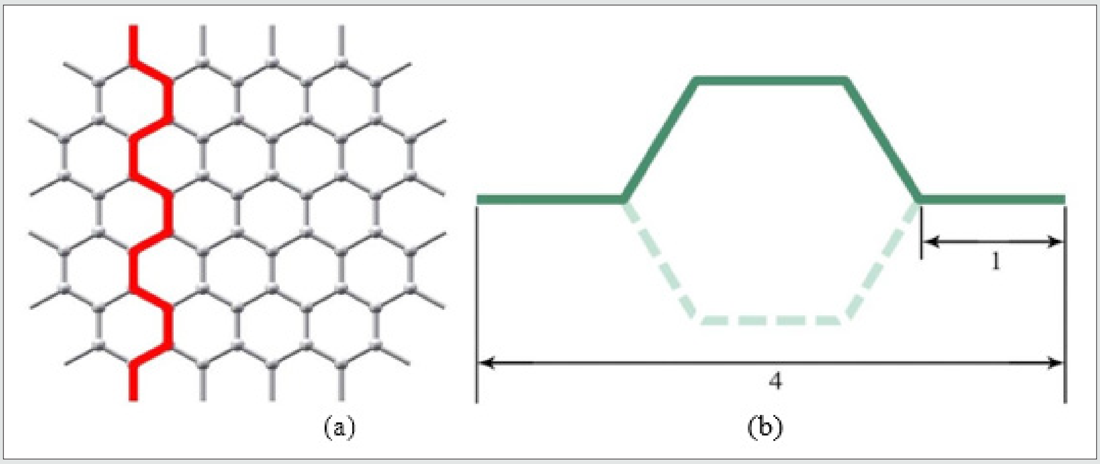

Figure 4: Graphene's geometry (a) and its unit (b).

According to Eq. (5), the resistance of a carbon nanotube can be expressed as

$$R = \mu L^{2.5} \quad (11)$$

where μ is a constant, L is the length of the lead.

Experiment

In our experiment, Chung Hwa Drawing Pencils (China First Bengbu Pencil Co., LTD.) with 2B type and the Fluke insulation resistance measuring instrument (Model no. 1508, Fluke Corporation, USA) were used to measure the lead's resistance with respect to its length, see Figure 5. The experimental data were given in Table 1. Consider the contact resistance in the measuring process, we write Eq. (11) in the form

$$R = R_0 + \mu L^{2.5} \quad (12)$$

where R_0 is the contact resistance.

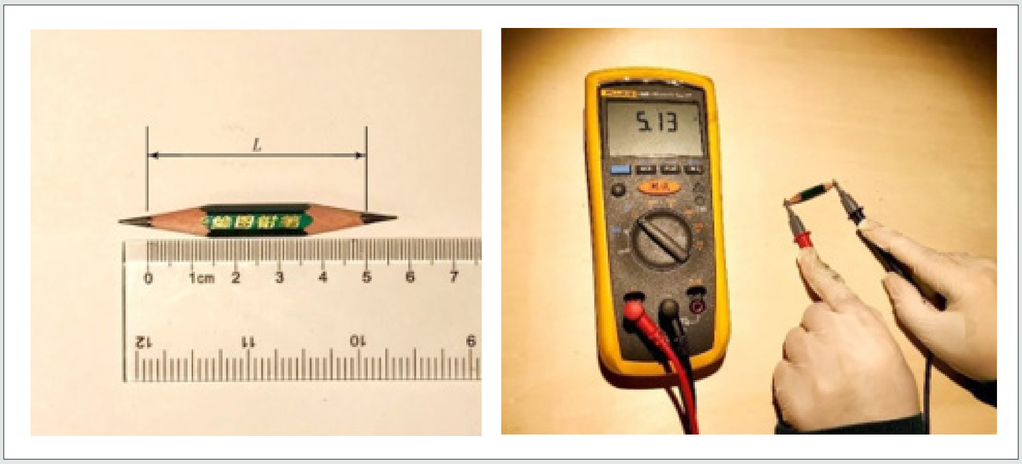

Figure 5: Pencil and insulation resistance meter for experiment.

Table 1: Experimental data.

Lead's length, L(cm)	Resistance, R(Ω)
5	5.13
6	5.26
7	5.35
8	5.72
9	5.74
10	6.19
11	6.64
12	6.95
13	7.4
14	7.88
15	8.59
16	8.79
17	9.74

By the least square method, using the experimental data given in Table 1, we can identify

$$\mu=0.004,\ R_0=4.9048 \quad (13)$$

So, the lead's resistance for our experiment is

$$R = 4.9048 + 0.004L^{2.5} \quad (14)$$

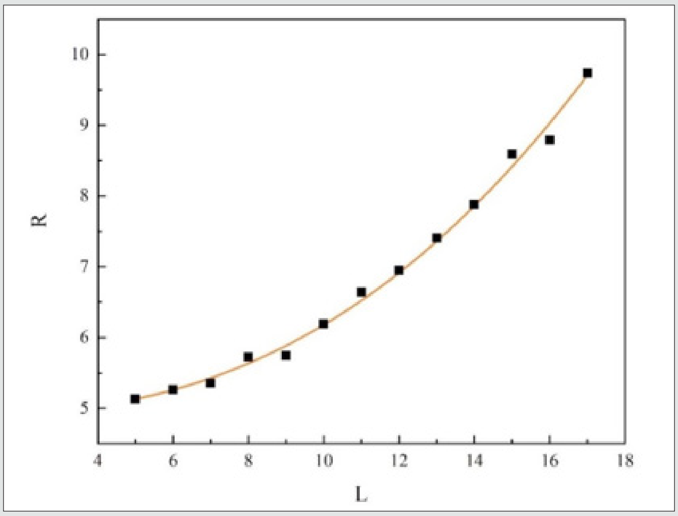

Figure 6: Resistance (Ω) versus length (cm) for a pencil's lead. The dots are experimental data, the continuous line is the theoretical prediction by Eq. (14).

The theoretical prediction sees good agreement with the experimental data as shown in Figure 6. Figure 6 Resistance (Ω) versus length (cm) for a pencil's lead. The dots are experimental data, the continuous line is the theoretical prediction by Eq. (14).

Conclusions

This paper uses a widely used carbon-based material for pencils to verify the scaling law of the resistance, which can be applied for wearable fabric sensors for various applications especially in smart fabrics. The experimental results verify that pencil's lead consists mainly of graphene with a fractal dimension of 2.5. This paper is helpful for study of the electronic property of graphene-based composites, especially, the SiC/graphene composites [14].

Conflict of Interest

The Author declares that there is no conflict of interest.

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