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Review Article

Semi-Balanced Array and CDC method (4)

MK Sharma* and Mohammed Omer

Department of Statistics, Addis Ababa University, Ethiopia

***Corresponding author:** M K Sharma, Department of Statistics, Addis Ababa University, Addis Ababa, Ethiopia

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Abstract

With a large number of lines in a complete diallel cross experiments method (4), the number crosses become unmanageable to be accommodated in homogeneous blocks. To overcome this problem incomplete block designs for complete diallel cross Griffing's method (4) with replication 2 and 1 have been proposed by using semi balanced arrays (p (p^{-1})/2, p, p, 2), where p is an odd prime or power of an odd prime. The analysis of data obtained through proposed design is presented. The analysis includes the analysis of variance, estimation of general combining ability and of specific combining ability. The analysis is illustrated with the help of numerical data. Table of universally optimal block designs has been provided.

Keywords: Orthogonal Array; Semi-Balanced Array; Complete Diallel Cross; Block Design; Optimality

Introduction

Rao (1973) [1] gave method of construction of semi-balanced array of strength 2. These arrays have been used in the construction of resolvable balanced incomplete block design. We are using these arrays in the construction of complete diallel cross (CDC) method (4)A common experimental design in genetics is the diallel, in which pairs of distinct lines are cross bred in order to estimate genetic effects [2]. Let p denote the number of lines, where p is an odd prime or power of odd prime, and it is desired to perform a diallel cross experiment involving v = 1/2p(p^{-1}) crosses of the type (i x j) between lines i and j , where i< j = 1,..., p. This is the CDC method (4) mating design with p inbred lines. [2] discussed in detail the analysis of this design in randomized block design (RBD). Later incomplete block designs were introduced by many authors See Singh et al. [3] to resolve the problem of estimating the genetic parameters precisely. The problem of generating optimal block designs for CDC method (4) for the case where treatment differences are due to only general combining abilities (gca) has been investigated by several authors such as [4-9]. [10] Considered the case when specific combining ability (sca) effects are also present in the model. They defined balanced and orthogonal designs i.e designs that are (i) balanced for gca comparisons (ii) balanced for sca comparisons and (iii) orthogonal in the sense that the least- square estimates of any gca contrasts is uncorrelated with the least- square estimate of any sca contrast. They gave a necessary condition for adesign to be balanced and orthogonal. They further proved that a design is balanced and orthogonal if and only if, it is a triangular partially balanced incomplete design (TPBIB design). Choi et al. [11] showed that the several series [4^{-1}2] of designs, which are optimal for gca comparison, are also optimal when sca effects are also present in the model although they are not TPBIB design. The above series designs have only one replication for each cross. Choi et al. [11] also reported three series of new designs in which each cross is replicated only once. So due this drawback all the sca effects are not estimable in these designs. This necessitates the construction of optimal block designs for CDC method (4) by which one can estimate both gca and sca effects uncorrelated to each other from the model.In the present paper, we are deriving optimal incomplete block designs for CDC methods (4) through semi balanced arrays. From these designs we can derive two more design for CDC methods (4) by dividing the blocks into two groups, provided the total number of blocks is even. The above designs are found to be optimal in the sense of [13,14].The rest of this article is organized as follows: in section 2, we give some definitions. In section 3, we discuss Lemma without proof given by [15]. In section

4, we discuss the relation of semi-balanced array with designs for CDC method (4) with example. In section 5, we give method of analysis of these designs. In section 6, we discuss optimality of proposed design and efficiency factor in comparison to randomized block design. In section 6 and, we have explained the analysis with the help of numerical data and discussed results obtained from the example.

Some Definitions

a. Definition: According to [15], a (N, r, p) array is said to be a semi-balanced array of strength d if for any selection of d rows α_1, $\alpha_2, \ldots, \alpha_d$, we denote d rows by $n(i_1, i_2, \ldots, i_d)$.

(i) $n(i_1, i_2, \ldots, i_d) = 0$ if any two i_j are equal.

(ii) $\sum_s n(i_1, i_2, \ldots, i_d) = \lambda$ constant

Where s represents summation over all permutation of distinct elements $i_1, i_2, \ldots, i_d$.

b. Definition: According to [4], a diallel cross design to be orthogonally blocked if each line occurs in every block r/b times, where r is the constant replication number of the lines and b is the number of blocks in the design.

Method of Construction of Semi-Balanced Array

A semi balanced array of strength 2 is useful in the construction of some combinatorial arrangements. The minimum value of N in such case is $p(p^{-1})/2$ with $\lambda = 1$. [15] gave following Lemma which gives the construction of semi balanced array $(p(p^{-1})/2, p, p, 2)$ and also has maximum number of rows.

Lemma: Let p be an odd prime or an odd prime power in which case there exists a Galois field with s elements, GF(s). Then $(p(p^{-1})/2, p, p, 2)$ semi balanced array exists.

For proof, see Rao(1973)

Relation between Semi-balanced Array $(p(p^{-1})/2, p, p, 2)$ and Designs for CDC System

Consider a semi-balanced array $(p(p^{-1})/2, p, p, 2)$, where p is an odd prime or power of odd prime. There are $(p^{-1})/2$ total sets in a semi-balanced array $(p(p^{-1})/2, p, p, 2)$. If we identify the elements of semi- balanced array as lines of a diallel cross experiment and ignoring first row of each set perform crosses in any two sets among the corresponding columns appearing in the same two sets, we get a mating design for diallel cross experiment involving p lines with $v = p(p^{-1}/2)$crosses, each replicated twice OR ignoring first row of each set and super imposing any two sets, we may also get a mating design for diallel cross experiment involving p lines with $v = p(p^{-1}/2)$crosses, each replicated twice. Now consider rows as blocks, we obtain block design d for Griffing's CDC method (4) with parameters $v = p(p^{-1})/2$, $b = p^{-1}$, $k = p$, $r = 2$. From this design we can derive two more designs for CDC experiment method (4) with parameters $v = p(p^{-1})/2$, $b = (p^{-1})/2$, $k = p$, $r = 1$ by dividing the blocks in two groups containing first $(p^{-1})/2$ blocks in one group and last $(p^{-1})/2$ blocks in second group, provided the number of blocks should be an even integer.

Thus using the above procedure, we may obtain $\binom{(p-1)/2}{2} = \frac{(p-1)(p-3)}{8}$ and $\frac{(p-1)(p-3)}{4}$ different layouts designs for Griffing's CDC method (4). In block designs for Griffing's CDC method (4) each line occurs in every block r/b times and every cross is replicated twice. According to Gupta et al. (1995) these designs are orthogonally blocked. In an orthogonal design no loss of efficiency on the comparisons of interest is incurred due to blocking. A block design for which $\mathbf{N} = \theta\ \mathbf{1p\ 1'b}$ is orthogonal for estimating the contrasts among gca parameters, where **N** denotes the line versus block incidence matrix and θ is some constant.Choi et al [11] proved that orthogonally blocked designs remain optimal for the estimation of gca comparisons even in the presence of sca effects in the model when each cross is replicated twice. Designs for method (4) are balanced for gca comparisons

Theorem: The existence of Semi -Balanced Array $(p(p^{-1}), p, p, 2)$ implies the existence of $\frac{(p-1)(p-3)}{8}$ orthogonally blocked incomplete block design with parameters $v = p(p^{-1})/2$, $b = p^{-1}$, $k = p$ and $r = 2$ for [2]method (4).If the number of blocks is an even integer, then we may obtain $\frac{(p-1)(p-3)}{4}$ orthogonally blocked incomplete block design with parameters $v = p(p^{-1})/2$, $b = (p^{-1})/2$, $k = p$ and $r = 1$ for [2]method (4).

Example 5.1; The present example of semi-balanced array for p =5 has been taken from Rao (1973)**.** If p =5, the residue classes 0, 1, 2, 3, 4 mod (5) form a field. We write the 5 elements of GF (5) as 0, ±1, ±2, and hence the key sets are.

(0, 1, 2, 3, 4), (0, 2, 4, 1, 3) (5.1)

second vector is obtained from the first on multiplying by 2. Writing (4.1) vertically and generating the other columns by the addition of elements GF (5). We obtain 10 columns as shown below which is divided into two groups.

Group 1					**Group 2**				
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	1	2	3	4	0	1	2	3	4
1	2	3	4	0	2	3	4	0	1
2	3	4	0	1	4	0	1	2	3
3	4	0	1	2	1	2	3	4	0
4	0	1	2	3	3	4	0	1	2

From the semi balanced array given above, we may obtain designs for Griffing's CDC method (4) (i) by ignoring the first row

and perform crosses in any two sets among the corresponding column appearing in the same two sets and developing columns mod (5) we get a mating design for diallel cross experiment OR superimposing one set to over other set. This mating design can be transformed into environmental design with parameters v =10, b =4, k =5 and r =2, by considering rows as blocks as given below.

Design for Griffing's method (4)

B_1	1×2	2×3	3×4	4×0	0×1
B_2	2×4	3×0	4×1	0×2	1×3
B_3	3×1	4×2	0×3	1×4	2×0
B_4	4×3	0×4	1×0	2×1	3×2

From the above design, we may get two more incomplete block designs for CDC experiment method (4) with parameters v =10, b =2, k =5 and r =1. Taking blocks 1 and 2, we may get first design and taking blocks 3 and 4, we may get second design.

Analysis

For the analysis of data obtained from design d,we will follow two stage procedures for estimating gca and sca effects [16,17]. The first stage is to consider to estimate cross effects, $\tau = (\tau_{12}, ..., \tau_{p(p-1/2)})^T$ by the following linear model.

$$y = \mu 1 + X\tau + D\beta + e \tag{6.1}$$

where **y** be $n\times1$ vector of observations, **1** is the $n\times1$ vector of ones, **X** is the $n\times v$ design matrix for treatments and **D** is an $n\times b$ design matrix for blocks, that is, the (h,u)th((h,l)th) element of **X** (respectively, of **D)** is 1 if the hth observation pertains to the uth cross (to lth block),and is zero otherwise (h = 1, ..., n; u = 1, ..., v ; and l = 1, ... , b), μ is a general mean, τ is a $v\times1$ vector of treatment parameters, β is a $b\times1$ vector of block parameters and **e** is an $n\times1$ vector of residuals. It is assumed that vector β is fixed and **e** is normally distributed with $E(e) = 0$, $V(e) = \sigma^2 I$, where **I** is the identity matrix of conformable order. Following[18,19], the analysis of our proposed designs leads to the following reduced normal equations for the crosses for model (6.1).

$$C_d\tau = Q_d \tag{6.2}$$

Where $C_d = R - NK^{-1}N'$

And$\mathbf{Q}_d$= (Q_{1d},...., Q_{vd}) **= T - N K^{-1} B**

In the above expressions **R** and **K** are diagonal matrices of order v × v and b × b with common diagonal elements 2 and p, respectively. **N = X´D** is the v × b incidence matrix of the designs d; **T = X´ Y** and **B = D´Y** are vectors of cross totals and block totals of order v×1 and b × 1 respectively for designs d.

Hence a solution to (6.2) is given by

$$\hat{\tau} = C_d^{-} Q_d \tag{6.3}$$

where C_d^- is the generalized inverses of C_d with the property that$\mathbf{C\,\bar{}\,C = C}$. The sum of squares due to crosses are $\mathbf{Q'_d C_d^- Q_d}$withd.f. = rank ($\mathbf{C_d}$) for designs d with the expectation and variance of$\mathbf{Q}_d$ being

$$E(Q_d) = C_d\tau \quad V(Q_d) = \sigma^2 C_d \tag{6.4}$$

Now we will utilize the above equations to estimate the genetic parameters in the proposed design.

The second stage is to utilize the fact that the cross effects can be expressed in terms of gca and sca abilities. So we can write effects as follows:

$$\tau_{ij} = g_i + g_j + s_{ij} \tag{6.5}$$

where g_i (g_j) is the gca for the ith (jth) parent and s_{ij} ($s_{ij} = s_{ji}$) is the sca for the cross between the ith and the jth parent (i< j =0, 1, ... , p^{-1}) and we also impose restrictions to solve equation (6.5) that is $\sum_i g_i = 0$, and $\sum_j s_{ij} + s_{ii} = 0$, for all i. In matrix notation equation (5.5) can be written as

$$\tau = Zg + s \tag{6.6}$$

where **Z** = (z_{ui}) (u = 1, 2, ..., v : i = 0, 1, ..., p^{-1}) is the cross and gca relation matrix.

z_{ui}= 1 if the uth cross has only one parent = i'

= 0, otherwise.

and **g** = (g_1, ..., g_p)^T and $s = \{s_{12}, s_{12}, ..., s_{p(p-1)/2}\}^T$.

Following the approach used in[20],the equation (5.6) can then be written as

$$C_d\tau = C_d Z g + C_{ds}$$

i.e. $E(Q_d) = C_d Z g + C_{ds}$ (6.7)

Since the matrix $\mathbf{C}_d$ is singular, we use the unified theory of least squares due to [15]and replace the usual true inverse of variance-covariance matrix by a g-inverse of $\mathbf{C}_d$. So the estimator of **g** is

$$\hat{g} = (Z'C_dC_d^{-}C_dZ)^{-}Z'C_dC_d^{-}Q_d$$

$$= (Z'C_dZ)^{-}Z'C_d\tau \tag{6.8}$$

Here the matrix $(Z'C_dZ) = 2(p-2)[I_p - \frac{1}{p}1p\Upsilon p]$ (6.9)

Therefore $\hat{g} = 1/2(p-2)Z'C_d\tau$ (6.10)

Hence $\hat{g} = H_1\tau$, where $H_1 = Z'C_d$

Now Var $(\hat{g})$ = $\mathbf{H}_1\mathbf{C}^{-}\mathbf{H}_1\sigma^2 = \sigma^2 / 2(p-2)\mathbf{I}_p$ (6.11)

Since the covariance matrix of $\hat{\mathbf{g}}$ is a constant time the identity matrix, the design d is balanced for the general combining abilities. Hence the proposed design d is variance balanced for general

combining ability effects. We thus have the following theorem.

Theorem: For a positive odd prime or power of odd prime >3, if there exits semi-balanced array (p (p-1)/2, p+1, p, 2), then there always exists a V-B incomplete block design for CDC experiment method (4).

Now substituting the estimate of **τ** and **g** from equations (6.3) and (6.9) in equation (6.6), we obtain the estimator of **s,** namely

$$\hat{s} = (C_d^{-} - 1/2(p-2)\mathbf{Z}\mathbf{Z}')\mathbf{Q}_d = (C_d^{-} - \frac{1}{2(p-2)}ZZ')C_d$$

$$= \mathbf{H}_2 \tag{6.12}$$

where $\mathbf{H}_2 = (C_d^{-} - \frac{1}{2(p-2)}ZZ')C_d$

$$\text{Var}(\hat{s}) = \mathbf{H}_2'C^{-}\mathbf{H}_2\sigma^2 \tag{6.13}$$

Since $\mathbf{H}_1\mathbf{1}_v = 0, \mathbf{H}_2\mathbf{1}_v = 0, \mathbf{H}_1\mathbf{H}_2' = 0$, we haverank $(\mathbf{H}_1) = p-1$ and rank $(\mathbf{H}_2) = v-p$.

It follows that **g** and **s** are represented by treatment contrasts which carry p-1 and v-p degrees of freedom, respectively and that the contrasts representing **g** are orthogonal to those contrasts which represent**s**. This means that the proposed design d allows for gca and sca effects to be estimated independently of each other.

The sum of squares due to gca and sca for design d are given by

$$SS(gca) = \mathbf{Q}'_d\mathbf{Z}(\mathbf{Z}'\mathbf{C}_d\mathbf{Z})^{-}\mathbf{Z}'\mathbf{Q}_d \tag{6.14}$$

$$SS(sca) = \mathbf{Q}_d'(C_d^{-} - 1/(p-2)\mathbf{Z}\mathbf{Z}')\mathbf{Q}_d \tag{6.15}$$

The ANOVA is then given in Table 1.

Table 1: Design d.

B1	B2	B3	B4
1×2 (4)	2×4 (12)	3×1 (4)	4×3 (12)
2×3 (8)	3×0 (3)	4×2 (7)	0×4 (8)
3×4 (6)	4×1 (9)	0×3 (7)	1×0(3)
4×0 (9)	0×2 (7)	1×4 (5)	2×1 (10)
0×1(4)	1×3(3)	2×0(9)	3×2(4)

Optimalityand Efficiency Factor

Since in design d every line is crossed with an equal number of different lines and the environmental design is equi-replicate, equi-block size (k< n) and binary, now using proposition of [14]to prove optimality of the proposed design, we have following theorem.

Theorem: Let be a block design for diallel cross, satisfying

(i) trace $(C_{d*}) = k^{-1}b\{2k(k-1-2x) + px(x+1)\}$and

(ii) $C_{d*} = (p-1)^{-1}k^{-1}b\{2k(k-1-2x) + px(x+1)\}(\mathbf{I}p - p^{-1}\mathbf{1}p\mathbf{1}'p)$ is completely symmetric.

where = [2k/p],and for square matrix A, tr(A) stands for the trace. **I**p is an identity matrix of order p and **1**p**1**′p is a p × p matrix of all ones. Furthermore, using $d*\varepsilon D(p, b, k)$ all elementary contrasts among gca effects are estimated with variance $\left[2b^{-1}(p-1)k/\{2k(k-1-2x) + px(x+1)\}\right]\sigma^2$.Then d* is universally optimal in D (p, b, k) and in particular minimizes the average variance of the best linear unbiased estimators of all elementary contrasts among the general combining ability effects. Our proposed design d ε D (p, b, k) and has information matrix Cg = (Z′CZ) and its trace is equal to 2(p-1(p-2) which can be verified by equation (5.9) and it is equal to the trace given in theorem 6.1(i). Hence design d is universally optimal.Suppose that instead of the proposed design d, one adopts a randomized complete block design with 2 blocks, each block having all p (p-1)/2 crosses. [2] on page 473 shows that the variance of any elementary contrast among the gca effects is σ_1^2 / (p-2), where σ_1^2 is the per observation variance in the case of a randomized block experiment. It is clear from (6.10) that using design d each elementary contrast among gca effects is estimated with variance σ^2 / (p-2). . Hence the efficiency factor of design d compared to a randomized block design under the assumption of equal intra-block variances is given by

$$E = \frac{1/(p-2)}{1(p-2)} = 1 \tag{6.1}$$

It implies that design d is as efficient as randomized block design.

Illustration

We show the essential steps of a CDC experiment method (4), using an incomplete block design d. For this purpose, we take data from an experiment on the number of tillers per plant in pearl millet, reported by Sharma (1998) on page 220. The author used a randomized complete block design with v = 36 as he considered p^2 possible crosses including selling and reciprocal crosses, among p = 6 inbred lines. For the purpose of illustration, we take the data of relevant crosses from this experiment. The layout and observations in parentheses are given below [21]. There are10 crosses and each cross is replicated twice The ANOVA and the estimates of the gca and sca effects are given in Tables 2, 3, & 4.

The following exhibits the vectors of block totals B, cross totals T and adjusted block totals Q.

B = (31.00, 34.00, 32.00, 37.00)′

T = (7, 16, 10, 17, 14, 7, 14, 12, 19, 18)′

Q = (-6.60, 2.80, -3.20, 3.40, 0.40, -6.20, 0.80, -1.6, 5.80, 4.40)

Table 2: Analysis of variance for design d.

Source of variation	Degrees of Freedom	Sum of squares
Block	p-1	B′ B/p – G 2/ p(p-1)
Crosses (adjusted for blocks)	rank (Cd)	Q′d Cd¯ Q d
Gca	rank (H1)	Q̀d Z (Z′ Cd Z) ¯ Z′ Qd
Sca	rank (H2)	Qd′(Cd ¯ -1/2 (p+2) Z Z′) Qd
Residual	by subtraction	by subtraction
Total	n-1	y′y - G 2/ p(p-1)

Table 3: Analysis of variance of the data on number of tillers.

Source of variation	D.F	Sum of squares	Mean squares	F
Block	3	4.2		
Crosses (adjusted for blocks)	9	84.2	9.35	0.048 (NS)
GCA	4	75.53	18.88	0.68(NS)
SCA	5	8.466	1.69	
Residual	7	191.8	27.4	
Total	19	280.2		

Table 4: Estimates of the general combining ability effects and their estimated error on number of tillers.

Parent	Estimates of gca	± SE
1	-0.6	0.408
2	-1.93	0.408
3	1.23	0.408
4	-1.1	0.408
5	1.4	0.408

Result and Discussion

a. Variances: The simple ANOVA (Table2) tends to render information about the basic nature of the parental material involved in diallel crossing. Thus, significant variation among parents, among crosses and among F1's does indicate potential genetic differences among the parents chosen and their potency. It is clear from Table 1that the effect of crosses is not significant.

b. Effects: (i) General combining ability (gca) effects: The gca effects (gi) represent the additive nature of gene action. A high general combiner (parent) is characterized by its better breeding value when crossed with a number of parents. Depending upon the character concerned, the nature and magnitude of gi are both concerned. In the present example, none of the gi are significant (Table 5, 6).

c. Specific combining ability (sca) effects: The sca effects (sij) signify the role of non-additive gene action in character expression. The highly specific combining ability effects leading to highest performance of some specific cross combinations. In the present example sca effects are not significant.

Table 5: Estimates of the specific combining ability effects and their estimated error on number of tillers.

Sca	Estimates of sca	± SE	sca	Estimates of sca	± SE
S01	0.76	0.444	S13	-0.06	0.444
S02	0.76	0.444	S14	-0.06	0.444
S03	0.1	0.444	S23	-0.93	0.444
S04	-0.1	0.444	S24	0.-0.73	0.444
S12	0.9	0.444	S34	0.9	0.444

Table 6: Universally Optimal Block Designs for Diallel Cross method (4) experiments obtainable from semi-balanced arrays (p (p^{-1})/2, p, p, 2).

S.No.	p	v	b	k	r	Remark
1	4	6	3	4	2	New
2	5	10	2	5	1	New
3	5	10	4	5	2	New
4	8	28	7	8	2	New
5	9	36	4	9	1	New
6	9	36	8	9	2	New
7	16	120	15	16	2	New

Conclusion

Diallel crossing as mating designs are used to study the genetic properties of inbred lines in plant and animal breeding experiments. In the present article we have given some new incomplete block designs for Griffing's CDC method (4) by using semi –balanced array (p (p^{-1}), p, p, 2).

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