Some Simple Mathematical Models in Epilepsy

E Ahmed*
Mathematics Department, Faculty of Science, Mansoura, Egypt

*Corresponding author: E Ahmed, Mathematics Department, Faculty of Science, Mansoura, Egypt

Received: December 10, 2018
Published: December 17, 2018

Abstract
Epilepsy is a chronic disorder of the brain that affects people of all ages. Here epileptic seizures equations are related to the telegraph equation.

Introduction
Some Basic Facts about Epilepsy
Epilepsy is a chronic disorder of the brain that affects people of all ages. Approximately 50 million people worldwide have epilepsy, making it one of the most common neurological diseases globally. People with epilepsy respond to treatment approximately 70% of the time. About three fourths of people with epilepsy living in low- and middle- income countries do not get the treatment they need. Therefore, using different approaches to study epilepsy is useful [1].

2.2. Telegraph Equation in Epilepsy
Telegraph equation [2,3] is a generalization of diffusion equation where the standard diffusion equation depends on the continuity equation and Fick’s law

\[ j(x,t) = -D \frac{\partial c}{\partial x} \]  \hspace{1cm} (1)

Where,

\( j \) is the diffusing object (e.g. technology, concept, etc...),
\( c \) is the distribution function of this object and
\( D \) is the diffusion constant.

The resulting standard diffusion equation is

\[ \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \]  \hspace{1cm} (2)

where \( \partial \) is the second order differentiation w. r. t. position \( x \).

A basic weakness of this equation is that the flux \( j \) reacts simultaneously to the gradient of \( c \) and consequently an unbounded propagation speed is assumed. To solve this problem Fick’s law is replaced by

\[ j + T \frac{\partial j}{\partial t} = D \frac{\partial c}{\partial x} \]  \hspace{1cm} (3)

where \( T \) is a time constant which measures the memory or delay effect. Thus, one gets the telegraph equation:

\[ T \frac{\partial^2 c}{\partial t^2} + (1-T \frac{\partial^2 c}{\partial x^2}) \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} + f(c) \]  \hspace{1cm} (4)

Applying this equation to the case

\[ c(x,t) = \alpha e^t + \beta e^{-t} + \gamma \]  \hspace{1cm} (5)

and making the identifications

\[ c(t,x) = \sin(\lambda x) g(t), \quad T = 1/(\gamma^2), \]
\[ 2/T - c_2 t = 1 - T \alpha, \quad \alpha - T \gamma = c_1 + c_2 \]

one regains equations 1-4 in [4] where \( t_0 \) is small and we made the expansion

\[ g(t-t_0) \sim g(t) - t_0 \frac{dg}{dt} \]

These equations are used to explain dynamical properties of epileptic seizures.

References