Epilepsy Circle Map and Drug Resistance

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Introduction

Epilepsy is a chronic disorder of the brain that affects people of all ages. Approximately 50 million people worldwide have epilepsy, making it one of the most common neurological diseases globally.

A modified circle map has been proposed to describe epilepsy [1]

\[ X(t+1) = X(t) + c - r[\sin(\sqrt{X(t)})]^2, \]

where c and r are positive constants. The equilibrium solution is

\[ \sin^2(\sqrt{X}) = \frac{c}{r}, \]

It is stable if

\[ 2 > \left[ c(c-r) / X \right]^{(5)} > 0 \]

Contrary to the familiar circle map the MCME is Not periodic.

For the case of x(t) small such that \( \sin(x) \approx x \) it reduces to

\[ X(t+1) \approx (1-r)X(t) + c \]

Its winding number is \( \sim c-r \sum \frac{x(j)}{n} \).

Hence Arnold tongue exist so long as the winding number is rational. Here the following proposition is used:

Proposition: If \( f(x) \) is continuous and monotonic then the winding number of the system \( X(t+1) = f(X(t)) \) exists and is independent of the initial point.

Lyapunov exponent of (5) is \( \ln(1-r) \) i.e. the equilibrium solution of (5) is locally asymptotically stable.

A relation between epilepsy and telegraph equation has been derived in [2].

In epilepsy there is a competition between drug susceptible and drug resistant species [3]:

Recently [4], it has been pointed out that competition between different species is an important factor in drug resistance. Here we give a simple model for competition between susceptible and resistant species. We denote susceptible ones by S and resistant ones by R hence we have the model:

\[ \frac{ds}{dt} = aS(1-S) - SR, \frac{dR}{dt} = bSR - dR \]

Where a, b and d are positive constants. It is direct to see that the coexistence solution

\[ s = d / b, R = a(1-S), d / b < 1 \]

is locally asymptotically stable. This agrees with observations [5].

References
