



The Spectral Characterization of Hamiltonicity of Graphs

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Introduction

It is an important NP-complete problem in structure graph theory to judge whether a graph is Hamiltonian. So far, there is no perfect description on this problem. Therefore, it has always been concerned by the workers of graph theory and mathematics. It is explored that the new method for characterization of Hamiltonicity of graphs. Because the spectrum of a graph can well reflect the structural properties of a graph and is easy to calculate, at the 2010 conference of the theory of graph spectra, M. Fiedler and V. Nikiforov formally proposed whether the theory of graph spectra can be used to study the Hamiltonicity of a graph, and they [1] gave sufficient conditions for given graph to be Hamiltonian (or traceable) in terms of the spectral radius of the graph. Since then, relying on the spectrum of matrix representation of graph, giving the spectral sufficient conditions of the Hamiltonian graph has been a new method to study the Hamilton problem. Many results have been obtained by using the spectral radius and the signless Laplacian spectral radius of the graph to describe the Hamiltonicity of the graph. [2] firstly gave a sufficient condition for a graph G to be Hamiltonian and traceable by using the signless Laplacian spectral radius of the complement of the graph [3] optimized the condition of the number of edges of the Hamilton graph, gave a better condition for G to be traceable by using the spectral radius of the graph G , and firstly gave a sufficient condition for the balanced bipartite graph to contain the Hamilton cycle by the spectral radius of its quasi-complement graph. [4] firstly used the spectral radius and signless Laplacian spectral radius of the graph to describe the Hamilton-connected of the graph. [5] used the signless Laplacian spectral radius of the quasi-complement graph of the balanced bipartite graph to give a sufficient condition for the balanced bipartite graph to be Hamiltonian and used the signless Laplacian spectral radius of the graph G to give a sufficient condition for the graph G to be traceable or Hamilton-connected. [6] continued to study the relationship between the Hamiltonicity and spectral radius of general graphs and balanced bipartite graphs and extended the conclusions in [3] and [5]. [7] firstly proposed to use the stability of graphs to study the Hamiltonian properties of graphs, and also summarized the method of studying the

spectral characterization of the Hamiltonian graph by optimizing the boundary conditions of the Hamiltonian graph. [8] firstly characterized traceability of connected claw-free graphs by spectral radius. [9] discussed spectral conditions for Hamiltonicity of claw-free graphs. [10] firstly presented spectral sufficient conditions for a k -connected graph to be traceable or Hamilton-connected. [11] firstly presented sufficient conditions based on spectral radius for a graph to be k -connected, k -edge-connected, k -Hamiltonian, k -edge-Hamiltonian, β -deficient and k -path-coverable. Lately, [12] firstly gave spectral radius or signless Laplacian spectral radius conditions for a graph to be pancyclic.

Because the minimum degree of a graph is related to the density of the graph, with the deepening of research, people began to study the spectral characterization of Hamiltonian properties of graphs with large minimum degree conditions and gave the better conclusions. By adding the condition of large minimum degree, [13] firstly presented some (signless Laplacian) spectral radius conditions for a simple graph and a balanced bipartite graph to be traceable and Hamiltonian, respectively. Subsequently, [14] optimized the lower bound of the spectral condition of simple graphs with large minimum degree; [15] optimized the lower bound of the spectral condition of balanced bipartite graphs with large minimum degree. [16] characterized the signless Laplacian spectral radius conditions for a graph or balanced bipartite graph with large minimum degree to be Hamiltonian. [17] and [18] studied the signless Laplacian spectral radius condition for a graph with large minimum degree to be Hamilton-connected. [19] presented some spectral radius conditions for a balanced bipartite graph or a nearly balanced bipartite graph with large minimum degree to be traceable, respectively. [20] gave some spectral sufficient conditions for a balanced bipartite graph with large minimum degree to be traceable and Hamiltonian in terms of the spectral radius of the graph with large minimum degree. It strengthened the according results of Li and Ning for n sufficiently large. [21] presented some conditions for a simple graph with large minimum degree to be Hamilton-connected and traceable from every vertex in terms of the spectral radius of the graph or its complement respectively

and gave better conditions for a nearly balanced bipartite graph with large minimum degree to be traceable in terms of spectral radius, signless Laplacian spectral radius of the graph or its quasi-complement respectively. [22] presented sufficient spectral conditions of a connected graph with large minimum degree to be k -hamiltonian or k -path-coverable or β -deficient, for relatively large n . [23] gave the sufficient conditions for a graph with large minimum degree to be s -connected, s -edge-connected, β -deficient, s -path-coverable, s -Hamiltonian and s -edge-Hamiltonian in terms of spectral radius of its complement.

Other spectral characterizations on Hamiltonicity, at present, only [24,25] used the Laplacian eigenvalues to give the spectral sufficient condition for the graph to be Hamiltonian. [26] firstly applied the distance signless Laplacian spectral radius of the graph's complement to give a sufficient condition for the graph to be traceable or Hamiltonian. [27] firstly discussed the Hamiltonian in terms of the energy of graph. [28], by adding the maximum degree condition on the basis of [26], used the energy of complement graph to give the sufficient conditions for the graph to be traceable, Hamiltonian and Hamilton-connected. The results optimized the conclusions of [26] in a sense. [22] gave some sufficient conditions for a nearly balanced bipartite graph with large minimum degree to be traceable in terms of the energy, the first Zagreb index and the second Zagreb index of the quasi-complement of the graph, respectively. By observing, we find that the conditions of all the above conclusions deduce that the graphs are dense. In fact, there are many Hamiltonian graphs with non dense edge distribution, such as cycles. Therefore, it is necessary to study whether a non-dense graph (sparse graph) is a Hamiltonian graph, but there are few research results. Komlos and Szemerédi proposed that almost all graphs are Hamiltonian graphs in 1975. Inspired by this, [29] characterized the Hamiltonian property of regular graphs by using the adjacency spectrum of graphs; [30] characterized the Hamiltonian property of almost regular graphs by using the Laplace spectrum of graphs. In 2011, Radcliffe proposed whether we can give sufficient conditions for Hamiltonian graphs by using the normal Laplace spectrum of graphs. In 2012, [31] studied this problem, gave corresponding conclusions, and explained that the conclusions are applicable to the determination of Hamiltonicity of general graphs.

Although there are a lot of results, there are still many problems worthy of further study. Firstly, combining the ideas from theorems of Ore and Fan to develop extremal spectral conditions for dense graphs (with given connectivity, toughness, forbidden subgraphs) to be Hamiltonian (or related structural properties). Such as, finding spectral sufficient conditions for a graph or its complement to be Hamilton-connected, or k -Hamiltonian, k -path-coverable, k -edge-Hamiltonian; signless Laplacian spectral radius sufficient conditions for a (nearly) balanced bipartite graph or its nearly complement to be traceable, or Hamiltonian; (signless Laplacian) spectral radius sufficient conditions for the nearly complement of a balanced bipartite graph to be bipancyclic or considering the case with minimum degree; the characterization of Hamiltonicity

of graphs with 1-tough [32,33]. Secondly, due to the difficulty of research, there are only three papers to study Hamiltonian properties of sparse graphs, but there are many sparse Hamiltonian graphs. Therefore, there is a lot of space to explore some sufficient or necessary conditions for sparse graphs with some properties to be Hamiltonian (or related structural properties) by using the spectrum of the graph and corresponding eigenvector. At last, directed graphs also have Hamiltonian properties, but all previous studies have only considered undirected graph. So, it is also very valuable to study extremal spectral conditions for oriented graphs to be Hamiltonian. The research on the spectral characterization of Hamiltonicity of graphs build a bridge between structure graph theory and algebraic graph theory. Expected results not only enrich the study of Hamilton problem in structural graph theory, but also extend the spectral study of algebraic graph theory, thus promoting the research of algebraic method of Hamilton problem.

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